# Optimal Power Allocation for NOMA-based Cellular Two-Way Relaying

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## Abstract

This paper proposes a non-orthogonal multiple access (NOMA) based low-complexity relaying approach for multiuser cellular two-way relay channels (CTWRCs). In the proposed scheme, the relay detects the signal using successive interference cancellation (SIC) and re-generates the transmit signal with zero-forcing (ZF) transmit precoding. The achievable data rates of the NOMA-based multiuser two-way relaying (TWR) approach is analyzed. We further study the power allocation among different data streams to maximize the weighted sum-rate (WSR). We re-form the resultant non-convex problem into a standard monotonic program. Then, we design a polyblock outer approximation algorithm to sovle the WSR problem.The proposed optimal power allocation algorithm converges fast and it is shown that the NOMA-TWR-OPA scheme outperforms a NOMA benchmark scheme and conventional TWR schemes.

***Keywords:*** Two-way relay channels, NOMA, optimal power allocation

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## 1. Introduction

Non-orthogonal multiple access (NOMA) is an efficient multiple access technique to support massive connections in Internet of Things (IoT) networks [1][2]. In a typical power-domain NOMA system, multiple users' signals are assigned with different power levels, and user with stronger channel gain needs to decode signals of other weaker users before decoding its own signal using successive interference cancellation (SIC). Compared with conventional orthogonal multiple access (OMA) schemes, multiple usersin NOMA systems can share the same time/frequency resources to improvespectral efficiency.

Recently, NOMA has been combined with other transmission techniques, such as cooperative transmission and wireless relaying, to improve the throughput of wireless systems [3]-[5]. In [3], the authors proposed a two-user cooperative NOMA approach, in which the user with better channel quality forwards the decoded signal to the weak user. For wireless relay networks with a fixed relay station, a coordinated direct and relayed transmission (CDRT) scheme based on NOMA was proposed in [4], where the relay decodes and forwards the received NOMA data to the other user. A similar scheme was designed in [5], where amplify-and-forward (AF) relaying method was employed.

The spectral efficiency of these one-way relay NOMA networks could be further improved through the combination of NOMA and the well-known two-way relaying technique [6][7]. The results in [7] show that NOMA-based TWR transmission can improvethe energy efficiency greatly as compared with conventional OMA-based transmissions. NOMA has also been studied in other two-way relaying systems, such as multipair two-way relay channels (TWRCs) [8]-[10]. The authors in [9] proposed a physical-layer network coding (PLNC) based NOMA scheme for two-way information exchange between multiple pair of users, which can improve the spectral efficiency and outage performance.

In this work, we apply the NOMA technique to another TWR channel, i.e., the cellular two-way relay channel (CTWRC). In CTWRC, a relay is deployed to assist the transmission between a base station (BS) and multiple users. Typically, the information exchange in a time-divison duplex (TDD) CTWRC is completed in two phases. One major challenge in CTWRC is the inter-stream interference (ISI). For a CTWRC with $K$ users, there will be $2K$ data streams in the system. To suppress the ISI, several AF and decode-and-forward (DF) based multiuser relaying schemes have been proposed in [11]-[13]. In all these approaches, the relay uses multiple antennasto detect the received mixed signals. For instance, the multiuser interference is cancelled by zero-forcing (ZF) precoding in AF-based CTWRC [11]. However, the performance of such AF-based relaying scheme is usually poorer than DF relaying. To improve the rate performance of CTWRC, Yang in [13] designed a DF-based TWR scheme, where the relay detects and re-encodes the received signals using lattice coding. It should be noted that a larger number of antennas at the relay is required as compared with AF-based approaches. Typically, the number of antennas should be doubled for DF-based relaying. As shown in **Fig. 1**, for the CTWRC with $K$ users, $2K$ antennas are needed at the relay for DF relaying scheme to detect the received data, while for AF-based approaches, only $K$ antennas is required. Also, the implementation complexity of DF-based approaches are much higher than AF-based approaches. For instance, high-complexity lattice coding and decoding are required in [13].

To balance the performance and complexity, we propose a NOMA-based multiuser relaying approach with optimal power allocation (OPA) for the CTWRC, which is referred to as NOMA-TWR-OPA in the following. In the proposed NOMA-TWR-OPA scheme, the received signals at the relay can be decoupled into multiple parallel sub-channels using signal space alignment precoding and ZF equalization. The two data streams associated with the same user forms a NOMA pair in each sub-channel, and can be decoded by the relay using low-complexity SIC. In the NOMA-TWR-OPAscheme, the relay only needs $K$ antennas as those AF-based approaches, and the decoding complexity at the BS and the relay is much lower than the DF-based approach in [13].

We first analyze therate performance of NOMA-TWR-OPA, and then study the power allocation for transmit precoding, aiming to maximize the weighted sum-rate (WSR) of CTWRC. The formulated WSR maximization problem is non-convex. We reform it as a monotonic program (MP) and design a polyblock outer approximation based algotirhm to solve the WSR problem. Numerical results shows that the power aloocation algotirhm converges fast and the NOMA-TWR-OPA scheme outperforms a NOMA benchmark scheme and the existing AF relaying schemes.

## 2. System Model

**Fig. 1** shows the considered CTWRC with a BS, a relay $R$, and $K$ mobile users $U_1,\ldots,U_K$. No direct links exist between the users and the BS, hence a relay is deployed to assist the communication. Both the BS and the relay are multi-antenna nodes with $N$ antennas. The channel between the BS and the relay is denoted by $\boldsymbol{H}_{BR} \in \mathbb{C}^{N\times N}$, and that from user $k$ to the relay is denoted by $\boldsymbol{g}_{k,R} \in \mathbb{C}^{N\times 1}$. We also define $\boldsymbol{H}_{UR} = [\boldsymbol{g}_{1,R},\ldots,\boldsymbol{g}_{K,R}] \in {}^{N\times K}$ for notation simplicity. We assume time-division duplex (TDD) mode is adopted that two time slots are needed to complete the two-way data exchange. In slot I, all users and the BS send signals to the relay $R$. The relay node $R$ transmits signal to the $K$ users and the BS after proper processing in slot II. All the channels are assumed to be unchanged and reciprocal during the two time slots.

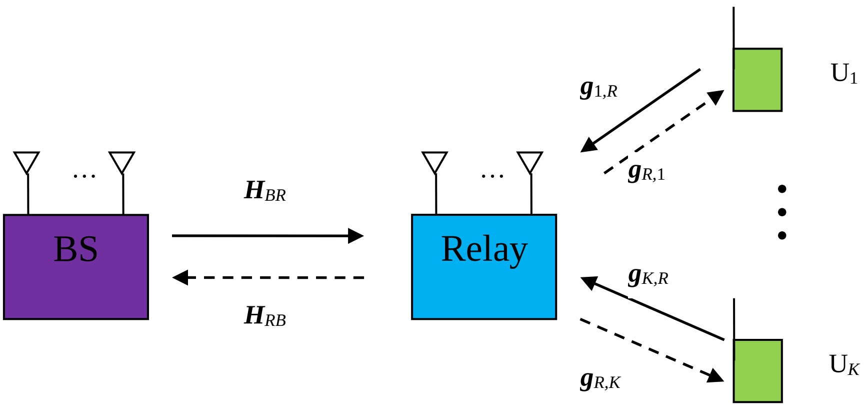


**Fig. 1.** NOMA-based two-way relaying for CTWRC.

## 2.1 The First Slot

The transmit signal of user $k$ is

$$x_{U,k} = \sqrt{p_{U,k}} s_{U,k}, \tag{1}$$

where $s_{U,k}$ is the unit-power symbol, and $p_{U,k}$ is the transmit power.

At the BS, let $s_{B,k}$ be the data foruser $k$, and define $\boldsymbol{s}_B = [s_{B,1},..., s_{B,K}] \in {}^{K\times 1}$. The transmit signal at the BS is generated by

$$\boldsymbol{x}_B = \boldsymbol{F}_B \boldsymbol{\Lambda}_B \boldsymbol{s}_B, \tag{2}$$

where $\boldsymbol{F}_B \in \mathbb{C}^{N\times K}$ is the precoding matrix, and $\boldsymbol{\Lambda}_B = \mathrm{diag}\left([\sqrt{p_{B,1}},....,\sqrt{p_{B,K}}]^T\right) \in \mathbb{C}^{K\times K}$ is an diagonal power allocation matrix. The BS precoding matrix $\boldsymbol{F}_B$ is given by

$$\boldsymbol{F}_B = \boldsymbol{H}_{BR}^{\dagger} \boldsymbol{H}_{UR}, \tag{3}$$

where $(\cdot)^{\dagger}$ denotes the pseudo inverse. The $\boldsymbol{F}_B$ in (3) is designed based on the signal alignment techqniue [11]. Note that $\boldsymbol{F}_B$ and $\boldsymbol{\Lambda}_B$ in (2) need to satisfy the BS' power constraint:

$$\mathrm{tr}(\boldsymbol{F}_B \boldsymbol{\Lambda}_B \boldsymbol{\Lambda}_B^H \boldsymbol{F}_B^H) \le P_B. \tag{4}$$

The received signal at the relay in the first slot is given by

$$\boldsymbol{y}_R = \boldsymbol{H}_{BR} \boldsymbol{x}_B + \boldsymbol{H}_{UR} \boldsymbol{\Lambda}_U \boldsymbol{s}_U + \boldsymbol{z}_R. \tag{5}$$

where $\boldsymbol{\Lambda}_U = \mathrm{diag}\left([\sqrt{p_{U,1}},...,\sqrt{p_{U,K}}]^T\right)$, $\boldsymbol{s}_U = [s_{U,1},..., s_{U,K}]^T$, and $\mathbf{z}_R \sim CN(\mathbf{0}, \sigma_R^2 \mathbf{I}_K)$ is the additive white Gaussian noise (AWGN).

Substituting (3) into (5), $\boldsymbol{y}_R$ can be expressed as

$$\boldsymbol{y}_R = \boldsymbol{H}_{UR}\left(\boldsymbol{\Lambda}_B \boldsymbol{s}_B + \boldsymbol{\Lambda}_U \boldsymbol{s}_U\right) + \boldsymbol{z}_R. \tag{6}$$

From (6), an uplink NOMA is constructed. To facilitate the decoding, the relay performs ZF equalization to $\boldsymbol{y}_R$ as $\tilde{\boldsymbol{y}}_R = \boldsymbol{H}_{UR}^{\dagger} \boldsymbol{y}_R$. From (6), the ZF-equalized signal $\tilde{\boldsymbol{y}}_R$ at the relay can be written as

$$\tilde{\boldsymbol{y}}_R = \boldsymbol{\Lambda}_B \boldsymbol{s}_B + \boldsymbol{\Lambda}_U \boldsymbol{s}_U + \tilde{\boldsymbol{z}}_R. \tag{7}$$

with $\tilde{\boldsymbol{z}}_R = \boldsymbol{H}_{UR}^{\dagger} \boldsymbol{z}_R$.

Note that both $\boldsymbol{\Lambda}_B$ and $\boldsymbol{\Lambda}_U$ are diagonal matrices. Hence, the signal transmission between the BS and the users are now decoupled in to $K$ parallel subchannels, as shown in (7). From (7), the equalized $k$-th subchannel signal is

$$\tilde{y}_{R,k} = \sqrt{p_{B,k}} s_{B,k} + \sqrt{p_{U,k}} s_{U,k} + \tilde{z}_{R,k}, \tag{8}$$

where $\tilde{z}_{R,k} \sim CN(0, \sigma_{R,k}^2)$.

From (8), we can see that the signal model for the $k$ -th subchannel is a typical two-user NOMA system. Hence, the relay is able to detect $s_{B,k}$ and $s_{U,k}$ using the SIC technique. As $p_{B,k}$ is usually much larger than $p_{U,k}$, the relay node first detects $s_{B,k}$ by treating $s_{U,k}$ as an interference. The, $s_{B,k}$ is removed from the signal $\tilde{y}_{R,k}$ to detect $s_{U,k}$.

## 2.2 The Second Slot

The relay node transmits its linear precoded signal $\boldsymbol{x}_R$ to the $K$ users and the BS in the second slot. The transmit signal $\boldsymbol{x}_R$ is generated as follows. The relay first performs the classic XOR network coding as

$$s_{R,k} = \hat{s}_{B,k} \bigoplus \hat{s}_{U,k}, k = 1,2,..., K, \tag{9}$$

where $\hat{s}_{B,k}$ and $\hat{s}_{U,k}$ denotes the detected signals for the $k$ -th user based on (8). Then, the transmit signal at the relay is generated as

$$\boldsymbol{x}_R = \boldsymbol{F}_R \boldsymbol{\Lambda}_R \boldsymbol{s}_R, \tag{10}$$

where $\boldsymbol{s}_R = [s_{R,1},..., s_{R,K}]^T \in {}^{K\times 1}$ , $\boldsymbol{\Lambda}_R = \mathrm{diag}\left([\sqrt{p_{R,1}},....,\sqrt{p_{R,K}}]^T\right)$ is a diagonal power allocation matrix, and $\boldsymbol{F}_R \in \mathbb{C}^{N\times K}$ is the linear precoding matrix. To facilitate the decoding at the users, we use ZF precoding and the matrix $\boldsymbol{F}_R$ is

$$\boldsymbol{F}_R = \boldsymbol{H}_{RU}^{\dagger}. \tag{11}$$

Note that the diagonal power allocation matrix $\boldsymbol{\Lambda}_R$ and the linear precoding matrix $\boldsymbol{F}_R$ in (10) should satisfy:

$$\mathrm{tr}\left(\boldsymbol{F}_R \boldsymbol{\Lambda}_R \boldsymbol{\Lambda}_R^H \boldsymbol{F}_R^H\right) \le P_R. \tag{12}$$

The received signals at user $k$ and the BS can be written as

$$y_{U,k} = \boldsymbol{g}_{R,k}^T \boldsymbol{x}_R + z_{U,k}, \tag{13}$$

and

$$\boldsymbol{y}_B = \boldsymbol{H}_{RB} \boldsymbol{x}_R + \boldsymbol{z}_B, \tag{14}$$

respectively, where $\boldsymbol{H}_{RB} \in \mathbb{C}^{N\times N}$ and $\boldsymbol{g}_{R,k} \in \mathbb{C}^{N\times 1}$ are the corresponding channel matrix (vector), and it is assumed that $\boldsymbol{H}_{RB} = \boldsymbol{H}_{BR}^T$ and $\boldsymbol{g}_{R,k} = \boldsymbol{g}_{k,R}$ in the considered TDD system. $\boldsymbol{z}_B$ and $z_{U,k}$ are the AWGN.

Recall that we use relay ZF precoding that $\boldsymbol{F}_R = \boldsymbol{H}_{RU}^{\dagger}$. Hence, we have $\boldsymbol{g}_{R,k}^T \boldsymbol{F}_R = \sqrt{p_{R,k}}$ , and $y_{U,k}$ in (13) can be expressed as

$$y_{U,k} = \sqrt{p_{R,k}} s_{R,k} + z_{U,k}. \tag{15}$$

Based on (15), user $k$ can directly detect $s_{R,k}$.

For the BS, it first performs ZFequalization to the received signal as $\tilde{\mathbf{y}}_B = \mathbf{W}_B^H \mathbf{y}_B$, where $\boldsymbol{W}_B^H = (\boldsymbol{H}_{RB}\boldsymbol{W}_T)^\dagger$. From (14), the equalized signal $\tilde{\mathbf{y}}_B$ is

$$\tilde{\boldsymbol{y}}_B = \boldsymbol{\Lambda}_R \boldsymbol{s}_R + \boldsymbol{W}_B^H \boldsymbol{z}_B. \tag{16}$$

From (16), the BS can directly detect $s_{R,k}, k = 1,2,..., K$, and then recover $s_{U,k}$ through XOR operation based on the knowledge of its own signal $s_{B,k}, k = 1,2,..., K$.

## 3. Achievable Rates

In the following, we will analyze the achievable data rates of the NOMA-TWR-OPA scheme.Consider the data transmission in the first slot. From (8), the achievable rate from the BS to the relay is given by

$$R_{BR,k} = \frac{1}{2}\log_2\left(1 + \frac{p_{B,k}}{p_{U,k} + \sigma_{R,k}^2}\right). \tag{17}$$

And that from user $k$ to the relay is given by

$$R_{UR,k} = \frac{1}{2}\log_2\left(1 + \frac{p_{U,k}}{\sigma_{R,k}^2}\right). \tag{18}$$

From (15), the data rate from the relay $R$ to user $k$ is

$$R_{RU,k} = \frac{1}{2}\log_2\left(1 + \frac{p_{R,k}}{\sigma_{U,k}^2}\right). \tag{19}$$

Similarly, from (16), the achievable rate from the relay to the BS foruser $k$'s signal is given by

$$R_{RB,k} = \frac{1}{2}\log_2\left(1 + \frac{p_{R,k}}{\sigma_{B,k}^2}\right), \tag{20}$$

where $\sigma_{B,k}^2 = \mathrm{Var}(\boldsymbol{w}_{B,k}^H \boldsymbol{z}_B)$.

From (17) and (19), the downlink data rate from the BS to user $k$ is

$$R_{B,k} = \min(R_{BR,k}, R_{RU,k}). \tag{21}$$

Similarly, from (18) and (20), the uplink rate from user $k$ to the BS is

$$R_{U,k} = \min(R_{UR,k}, R_{RB,k}). \tag{22}$$

## 4. Optimal Power Allocation

Power allocation is very important on the performance of NOMA systems. From (21) and (22), we see that the achievable rates $R_{B,k}$ and $R_{U,k}$ depend on $p_{B,k}$ and $p_{R,k}$. In this section, we will find the optimal $p_{B,k}$ at the BS and $p_{R,k}$ at the relay for the proposed NOMA-TWR-OPAscheme.

Define $\boldsymbol{P}_B = [p_{B,1},\ldots,p_{B,K}]^T$ , $\boldsymbol{P}_R = [p_{R,1},\ldots,p_{R,K}]^T$ , and $\boldsymbol{P} = [\boldsymbol{P}_B^T, \boldsymbol{P}_R^T]^T$ . The achievable rate $R_{B,k}$ in (21) can be written as

$$R_{B,k}(\boldsymbol{P}) = \frac{1}{2}\log_2(1+\gamma_{B,k}(\boldsymbol{P})), \tag{23}$$

where

$$\gamma_{B,k}(\boldsymbol{P}) = \min\left(\frac{p_{B,k}}{p_{U,k}+\sigma_{R,k}^2}, \frac{p_{R,k}}{\sigma_{U,k}^2}\right). \tag{24}$$

Similarly, $R_{U,k}$ in (22) can be written as

$$R_{U,k}(\boldsymbol{P}) = \frac{1}{2}\log_2(1+\gamma_{U,k}(\boldsymbol{P})), \tag{25}$$

where

$$\gamma_{U,k}(\boldsymbol{P}) = \min\left(\frac{p_{U,k}}{\sigma_{R,k}^2}, \frac{p_{R,k}}{\sigma_{B,k}^2}\right). \tag{26}$$

Note that the data streams of different users usually have different QoS. In this work, we aim to maximize the weighted sum-rate for the NOMA-TWR-OPA scheme. Let $w_{B,k}$ and $w_{U,k}$ denote the priority weights for $R_{B,k}(\boldsymbol{P})$ and $R_{\mathrm{U},k}(\boldsymbol{P})$ of user $k$. The weighted sum-rate is

$$R_{\mathrm{sum}}(\boldsymbol{P}) = \sum_{k=1}^{K}\left(w_{B,k}R_{B,k}(\boldsymbol{P}) + w_{U,k}R_{U,k}(\boldsymbol{P})\right), \tag{27}$$

and the optimization problem can be formulated as

$$\begin{aligned} \max_{\boldsymbol{P}} R_{\mathrm{sum}}(\boldsymbol{P}) &= \sum_{k=1}^{K}\left(w_{B,k}R_{B,k}(\boldsymbol{P}) + w_{U,k}R_{U,k}(\boldsymbol{P})\right) \\ \text{s.t.}\quad & \mathrm{tr}(\boldsymbol{F}_B\boldsymbol{\Lambda}_B\boldsymbol{\Lambda}_B^H\boldsymbol{F}_B^H) \le P_B \\ & \mathrm{tr}(\boldsymbol{F}_R\boldsymbol{\Lambda}_R\boldsymbol{\Lambda}_R^H\boldsymbol{F}_R^H) \le P_R \end{aligned} \tag{28}$$

The problem (28) is not a convex problem. In the following, we reformulate it as a classic monotonic program, and solve it using the polyblock method [14].

Define the weight vector $\boldsymbol{w} = [w_{B,1},\ldots,w_{B,K}, w_{U,1},\ldots,w_{U,K}]$ , and the SINR vector $\gamma_i(\boldsymbol{P}) = [\gamma_{B,1}(\boldsymbol{P}),\ldots,\gamma_{B,K}(\boldsymbol{P}), \gamma_{U,1}(\boldsymbol{P}),\ldots,\gamma_{U,K}(\boldsymbol{P})]$. The weighted sum-rate $R_{\mathrm{sum}}$ in (27) can be written as

$$R_{\text{sum}}(\boldsymbol{P}) = \sum_{i=1}^{2K} \frac{w_i}{2} \log(1+\gamma_i(\boldsymbol{P})). \tag{29}$$

where $w_i$ is the $i$-th element of $\boldsymbol{w}$.

Define the set

$$\mathcal{P} := \{\boldsymbol{P} \mid \text{tr}(\boldsymbol{F}_B \boldsymbol{\Lambda}_B \boldsymbol{\Lambda}_B^H \boldsymbol{F}_B^H) \le P_B, \text{tr}(\boldsymbol{F}_R \boldsymbol{\Lambda}_R \boldsymbol{\Lambda}_R^H \boldsymbol{F}_R^H) \le P_R\} \tag{30}$$

Introducing an auxiliary vector $\boldsymbol{z} = [z_1,\ldots,z_{2K}]^T$, we can rewrite (4) into

$$\max_{\boldsymbol{z}\in\mathcal{Z}} \Phi(\boldsymbol{z}) := \sum_{i=1}^{2K} \frac{w_i}{2} \log_2(z_i). \tag{31}$$

where $\mathcal{Z} := \{\boldsymbol{z} \mid 1 \le z_i \le 1+\gamma_i(\boldsymbol{P}), i=1,\ldots,2K, \forall \boldsymbol{P} \in \mathcal{P}\}$.

Let

$$\mathcal{A} := \{\boldsymbol{z} \mid 0 \le z_i \le 1+\gamma_i(\boldsymbol{P}), i=1,\ldots,2K, \forall \boldsymbol{P} \in \mathcal{P}\}. \tag{32}$$

Define $\mathcal{B} := \{\boldsymbol{z} \mid z_i \ge 1, \forall i\}$. Then, the feasible set $\mathcal{Z} = \mathcal{A} \cap \mathcal{B}$, and the problem (32) is a standard monotonic program as

$$\begin{aligned} &\max_{\boldsymbol{z}} \Phi(\boldsymbol{z}) \\ &\text{s.t. } \boldsymbol{z} \in \mathcal{A} \cap \mathcal{B}. \end{aligned} \tag{33}$$

For the problem (33), we can use a polyblock outer approximation approach to solve ti efficiently [14]. The main idea of this approach is to construct a nested sequence of polyblocks $\mathcal{P}_n$, $n=1,2,\ldots$, approximating $\mathcal{A} \cap \mathcal{B}$: $\mathcal{P}_1 \supset \mathcal{P}_2 \supset \cdots \supset \mathcal{A} \cap \mathcal{B}$ in such a way that $\max_{\boldsymbol{z}\in\mathcal{P}_n} \Phi(\boldsymbol{z}) \to \max_{\boldsymbol{z}\in\mathcal{A}\cap\mathcal{B}} \Phi(\boldsymbol{z})$.

A key step in the construction of the new outer polyblock $\mathcal{P}_{n+1}$ from $\mathcal{P}_n$ is to find the following projection by solving [14]:

$$\lambda^n = \max\{\alpha \mid \alpha \boldsymbol{z}^n \in \mathcal{A}\}, \tag{34}$$

where $\boldsymbol{z}^n = \arg\max_{\boldsymbol{z}\in\mathcal{T}_n} \Phi(\boldsymbol{z})$.

From (31), $\lambda^n$ can be determined as

$$\begin{aligned} \lambda^n &= \max\{\alpha \mid \alpha \boldsymbol{z}^n \in \mathcal{A}\} \\ &= \max\{\alpha \mid \alpha \le \min_i \frac{1+\gamma_i(\boldsymbol{P})}{z_i^n}, \forall \boldsymbol{P} \in \mathcal{P}\} \\ &= \max_{\boldsymbol{P}\in\mathcal{P}} \min_{i=1,\ldots,2K} \frac{1+\gamma_i(\boldsymbol{P})}{z_i^n}. \end{aligned} \tag{35}$$

The above problem (35) is a standard max-min fractional problem and can be solved efficiently. Let $\lambda^n$ be the solution to (35), then we are able to obtain $\boldsymbol{y}^n = \lambda^n \boldsymbol{z}^n$ and construct $\mathcal{P}_{n+1}$ from $\mathcal{P}_n$. As a result, the polyblock-based method can be implemented to find the optimal $\boldsymbol{z}^{\text{opt}}$ of (33).

The overall optimal power allocation algorithm for the proposed NOMA-TWR-OPA scheme is shown in the following.

**Algorithm 1 : Optimal Power Allocation**

**Initialize**: select an accuracy level $\varepsilon > 0$, let $n = 0$, and current best value $B = 0$. Initialize vertex set $T_0$ and the outer polyblock $P_0$.

**Repeat**:

1). Finds $\mathbf{z}^n \in T_n$ that maximizes $\Phi(\mathbf{z})$ and solve (4) to obtain $\lambda^n$ and $\boldsymbol{y}^n = \lambda^n \boldsymbol{z}^n$.

2). If $\mathbf{y}^n \in B$ and $\Phi(\mathbf{y}^n) > B$, then $B = \Phi(\mathbf{y}^n)$ and $\bar{\boldsymbol{z}} = \boldsymbol{y}^n$.

3). Let $\boldsymbol{z}^n(i) = \boldsymbol{z}^n - (z_i^n - y_i^n)\boldsymbol{e}_i, i = 1, \ldots, 2K$, where $z_i^n$ and $y_i^n$ are the $i$-th entry of $\boldsymbol{z}^n$ and $\boldsymbol{y}^n$, respectively. Calculate $T_{n+1} = [(T_n \setminus \{\mathbf{z}^n\}) \cup \{\mathbf{z}^n(i)\}] \cap B$.

4) Remove from $T_{n+1}$ any $\boldsymbol{v}_j \in T_{n+1}$ that satisfying $\Phi(\boldsymbol{v}_j) \le B(1+\varepsilon)$.

5). Set $n = n+1$.

**Until** $T_n$ is empty.

## 5. Numerical Results

In the simulation, the channels are assumed to be Rayleigh flat fading, and each element in $\boldsymbol{H}_{BR}$ or $\boldsymbol{g}_{k,R}$ is i.i.d. with unit variance. The powers of the users are the same: $p_{U,k} = p_U, \forall k$. Unless otherwise specified, there are $K = 4$ users in the CTWRC and the number of antennas is $N = 4$ for the BS and the relay node. The noise components are complex white Gaussian with zero mean and unit variance. We compare the proposed NOMA-TWR-OPAscheme with the AF-based scheme in [11] and a NOMA-based scheme in [15], which is labeled as “NOMA benchmark" in the figures.

**Fig. 2** shows the convergence behavior of Algorithm 1 for power allocation in NOMA-TWR-OPA. The powers of the relay and the users are fixed to be $P_R = 20$ dBm and $P_U = 0$ dBm, respectively. The rate weights are chosen as $w_{B,k} = w_{U,k} = 1, \forall k$, i.e., the sum-rate is plotted in the figure. It can be seen thatAlgorithm 1 converges very fast and 8-10 iterations is sufficient to determine the optimal $p_{B,k}$ and $p_{R,k}$.

**Fig. 3** compares the sum-rate of NOMA-TWR-OPA with the other two counterparts. The system parameters are the same as in **Fig. 2**. It can be seen that as the trasnmit power of the BS increases, the achievable sum-rate of NOMA-TWR-OPAalso increases, and the sum-rate performance of NOMA-TWR-OPA outperforms the NOMA benchmark scheme and the AF-based scheme. For example, about $13\%$ sum-rate gain over the AF scheme is observed when $P_B$ is 26 dBm.

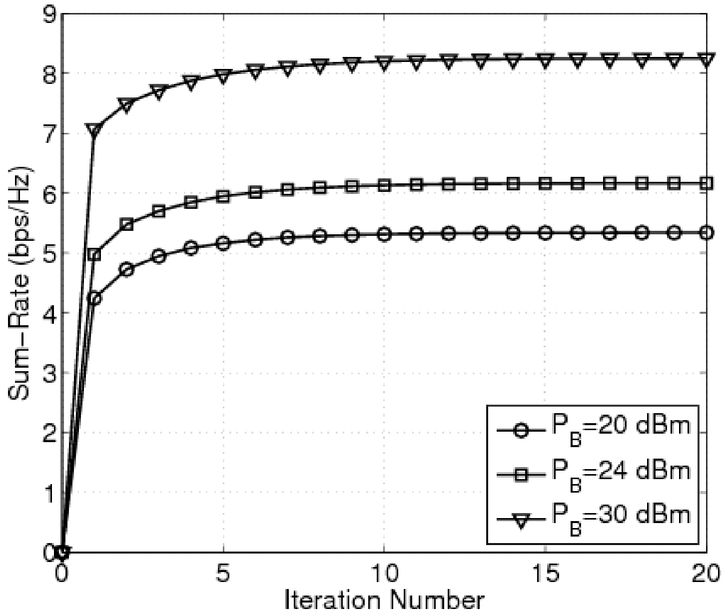


**Fig. 2.** Convergence behavior of the opimal power allocation algorithm with 4 users, $P_R$ =20 dBm, and $P_U$ = 0 dBm.

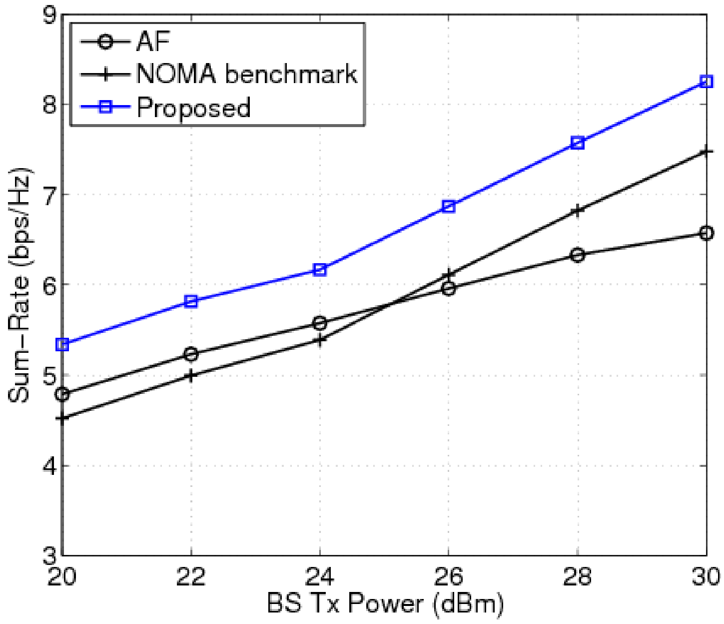


**Fig. 3.** Sum-rate performance comaprison with $K = 4$ users.

Next, **Figs. 4 and 5** show the weighted sum-rate and individual rate performance with different rate weights. It is assumed that the data transmission of user 1 has a higher priority, and the weights are chosen as $w_{B,1} = w_{U,1} = 2.5$, and $w_{B,n} = w_{U,n} = 0.5, n = 2,3,4$. From **Fig. 4**, we see that NOMA-TWR-OPA outperforms the counterpart schemes significantly, and the gap enlarges as compared with the equal weights case in **Fig. 3**. This is due to the fact that no power allocation was performed in these two schemes. While in the proposed scheme, the achievable rates of the users can be feasibly adjusted according to the rate weights and using the power allocation algorithm. As a result, the achievable rate of user 1 is much higher than the other three users as the weights for user 1 are much larger than the others, as shown in **Fig. 5**.

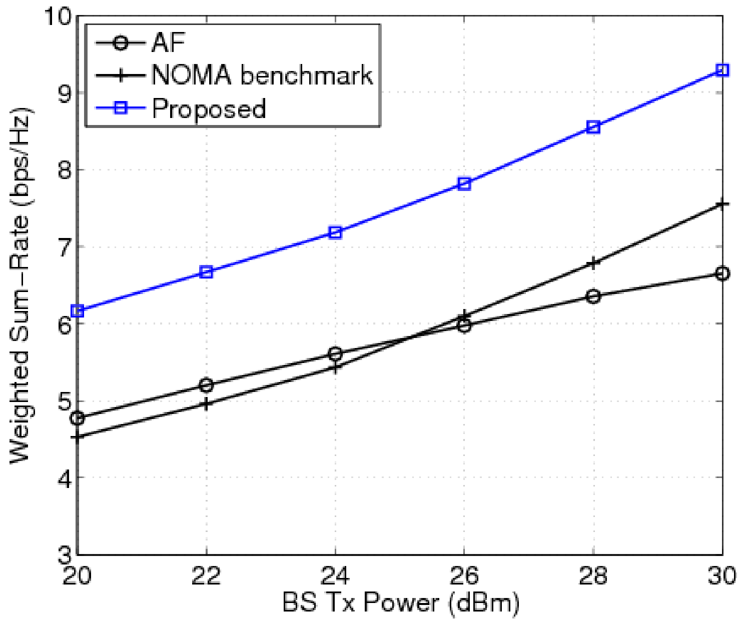


**Fig. 4.** Weighted sum-rate performance ofNOMA-TWR-OPA with $K = 4$ users.

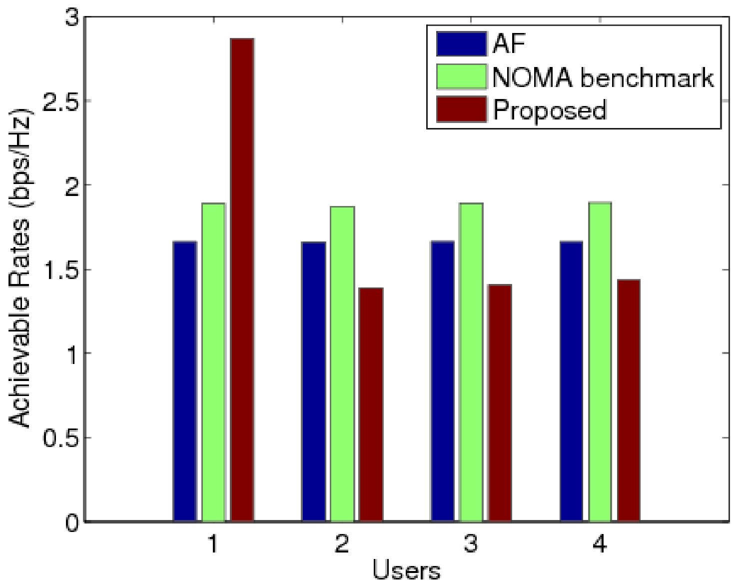


**Fig. 5.** Individual rate performance of NOMA-TWR-OPA with $K = 4$ users.

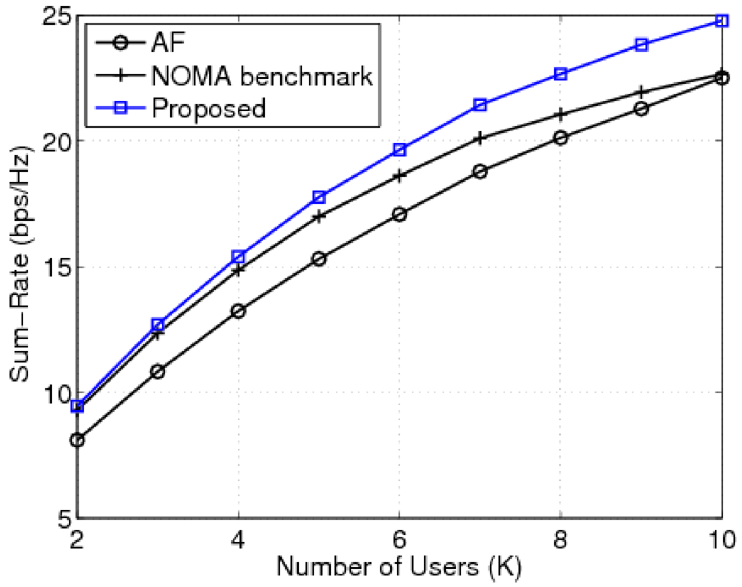


**Fig. 6.** Sum-rate versus the number of users, ( $P_B$ , $P_R$ , $P_U$ )=(40, 30,10) dBm.

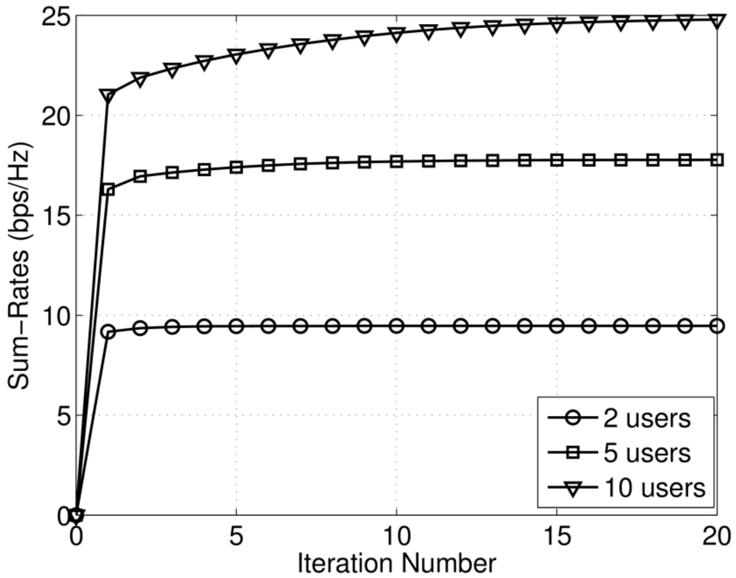


**Fig. 7.** Convergence behavior of Algorithm 1 with different number of users, ( $P_B$ , $P_R$ , $P_U$ )=(40, 30,10) dBm.

**Fig. 6** plots the sum-rate of NOMA-TWR-OPAwith different the number of users. While **Fig. 7** shows the corresponding convergence behavior of Algorithm 1 for power allocation. In both figures, the power budgets of the BS, the relay and the users are set to be 40, 30, and 10 dBm, respectively. During the simulation, we set $N$=$K$, i.e., the number of BS and relay antennas scales with the number of users. The weights are equal to one, i.e., the sum-rate performance is considered. From **Fig. 6**, it can be seen that NOMA-TWR-OPA achieves a higher sum-rate than the other two schemes, and the sum-rate gap enlarges as $K$ increases. From **Fig. 7**, we see that the proposed Algorithm 1 converges fast even with a large number of users. For system with 2 users, three iterations are sufficient. When there are 10 users, optimal power allocations can be found after 15 iterations.

## 6. Conclusion

A novel NOMA-based TWR scheme with optimal power allocation for CTWRCs was rpoposed, and the achievable data rates were analyzed. We studided the optimal power allocation to maximize the weighted sum-rate of CTWRC. The non-convex power allocation optimization problem was transformed into a MP problem, and was solved efficiently. Numerical results showed that 5-15 iterations are sufficient to find the optimal power allocation solution, and the proposed NOMA-TWR-OPA scheme outperforms the NOMA benchmark scheme without power allocation and the existing AF-based relaying scheme. The proposed scheme can be extended to MIMO CTWRC where the users are equipped with multi-antennas.

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